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**VARIABLE-SWEEP-RATE TESTING:  
A TECHNIQUE TO IMPROVE THE QUALITY  
AND ACQUISITION OF FREQUENCY RESPONSE  
AND VIBRATION DATA**

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# VARIABLE-SWEEP-RATE TESTING: A TECHNIQUE TO IMPROVE THE QUALITY AND ACQUISITION OF FREQUENCY RESPONSE AND VIBRATION DATA

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## SUMMARY

A technique has been established to improve the quality and reduce the acquisition time of frequency response and vibration data. The technique involves sweeping at a variable sweep rate which is slow during periods of dynamic importance and fast other times. A closed-loop controller senses system resonance and adjusts a basic sweep rate appropriately.

Fast sweeps, both linear and logarithmic, were used as base sweeps. The sweep rate was then reduced from the base sweep during resonance to improve data quality. Base sweep rates as high as 50 hertz per second were considered. With both base sweeps, significant improvements in sweep time, and data quality have been shown.

The technique was applied to a simulated lightly damped, doubly resonant system. Typical results for a linear base sweep showed a 7.5 to 1.0 time improvement while maintaining 1/2 percent accuracy. Conversely, maintaining a sweep time of 10 seconds improved the accuracy from a 35 to a 5.2 percent error.

## INTRODUCTION

Sweep-frequency methods of testing continue to be the mainstay methods for generating and collecting frequency response and vibration data. The sweep method provides a continuum of data as opposed to discrete frequency testing techniques. However, the price of this continuum is either (1) accuracy of results for a fixed sweep time, or (2) extremely long sweep times to obtain a desired level of accuracy.

The question of accuracy has been studied for linear sweeps by Reed, Hall, and Baker (ref. 1). They have generated plots of frequency shift and peak amplitudes and phase error as functions of sweep rate for a second-order system.

A technique for correcting slowly swept data, both logarithmically and linearly swept, has been evolved by Drain, Bruton, and Paulovich (ref. 2). Skingle (ref. 3) has considered the accuracy of applying correlation techniques to analyze the rapid frequency sweeps.

For the dynamic study of large systems (containing many channels of information), the testing problem is to obtain data of the desired accuracy in the minimum amount of time. This becomes very apparent, for example, when one considers vibration testing of a large missile system with hundreds of measurements. Reduction of such data, even by computer and even with such techniques as fast Fourier transform (FFT; see, for example, ref. 4), can take several hours for a single sweep. If the same accuracy of results can be maintained and the sweep time (and, hence, the analysis time) cut to 10, 25, or even only 50 percent of the original time, significant cost reductions could be effected.

One approach to this philosophy is to tailor the frequency sweep to the system being studied, such that the sweep is slow or nominal when something of dynamic importance is occurring and quite rapid otherwise. Those things of dynamic importance can be the system resonances, corner frequencies, nodes, etc. For this report the system resonances will be of interest.

An easy way to tailor the sweep rate to the system being studied is to have a closed-loop controller which senses system resonance and puts out an appropriate sweep rate.

Consideration of the nature of this controller and the improvements in accuracy and/or sweep time resulting from it are the primary subjects of this report. Two basic variable sweep-rate techniques will be considered: a linear base variable sweep and a logarithmic base variable sweep method.

## THE BASIC TECHNIQUE

The fundamentals of the technique involve (1) sensing the important system responses, and (2) controlling the sweep rate in some manner sensitive to these important responses.

Because, in this report, it has been assumed system resonances are the important dynamic phenomena, the sensing problem is that of determining system resonances as they occur. There are several possibilities for such sensing; however, the primary approaches are (1) absolute values of rate of change of amplitude ratio and (2) the negative rate of change of phase angle. The amplitude ratio and phase angle together with the rates are plotted in figure 1 for a typical second-order system. From this figure it can be seen that the phase angle or, more specifically, the rate of change of phase angle is a superior indicator of resonance. This is because it is insensitive to the amplitude level at which

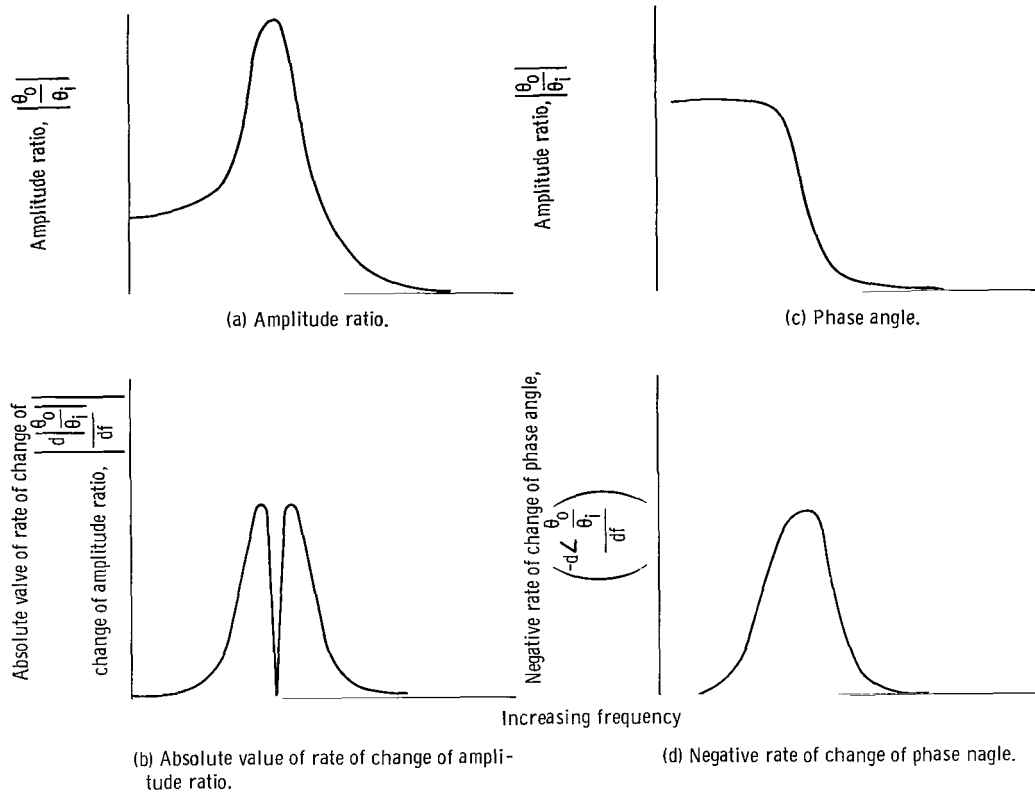


Figure 1. - Some variables of second-order system as functions of frequency.

resonance occurs and it is smooth going through the resonance, that is, no dip as in  $\left| \frac{d|\theta_o/\theta_i|}{df} \right|$ . If the amplitude level at which the peak occurs is important then  $|\theta_o/\theta_i|$  could be used as an indicator.

However, measurement of phase angle is quite difficult and requires fairly expensive equipment, but it is a relatively straightforward task to determine  $\left| \frac{d|\theta_o/\theta_i|}{df} \right|$ . For this reason  $\left| \frac{d|\theta_o/\theta_i|}{df} \right|$  is used as the indicator of resonance in this report.

The basic control diagram used in the study is shown in the simplified block diagram of figure 2. Here, basically only one item has been added to the conventional frequency response measurement setup and that is the controller which closes the feedback loop. Further, it is necessary to have an oscillator whose output frequency can be set by an external voltage.

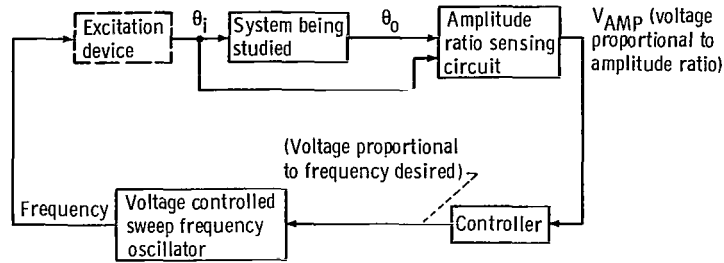


Figure 2. - Simplified diagram for variable sweep rate testing control.

The fundamental control concept is to sweep at a rate inversely proportional to  $\left| \frac{d|\theta_o/\theta_i|}{df} \right|$ . The amplitude ratio itself can also be used as an indicator, that is, feed-back as a control signal. This, then, would yield as a general method of control:

$$\frac{df}{dt} = \frac{K_0}{\left| \frac{d|\theta_o/\theta_i|}{df} \right|} - K_2 \left| \frac{\theta_o}{\theta_i} \right| \quad (1)$$

Since analog division poses scaling problems and practical problems in operation, the following similar techniques were used.

### Linear Variable-Sweep Technique

This technique is characterized by having a base sweep rate which is constant (i. e.,  $\dot{f} = \text{constant}$ ). From this base, the sweep rate is decreased as the magnitude of the rate of change of amplitude ratio or the amplitude ratio itself increases.

The derivative of amplitude ratio with respect to time is proportional to the derivative with respect to frequency; then the control law for the linear variable sweep is

$$\frac{df}{dt} = K_1 - K_0 \left| \frac{d|\theta_o/\theta_i|}{dt} \right| - K_2 \left| \frac{\theta_o}{\theta_i} \right| \quad (2)$$

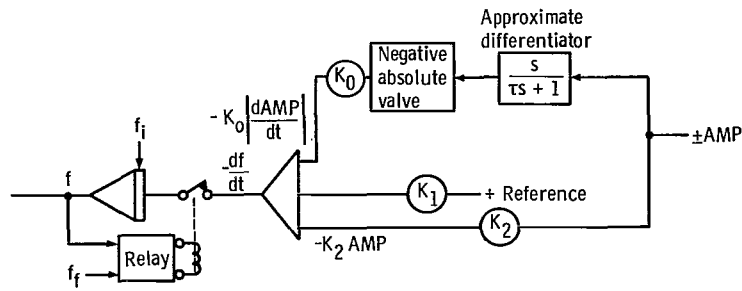


Figure 3. - Linear controller configuration.

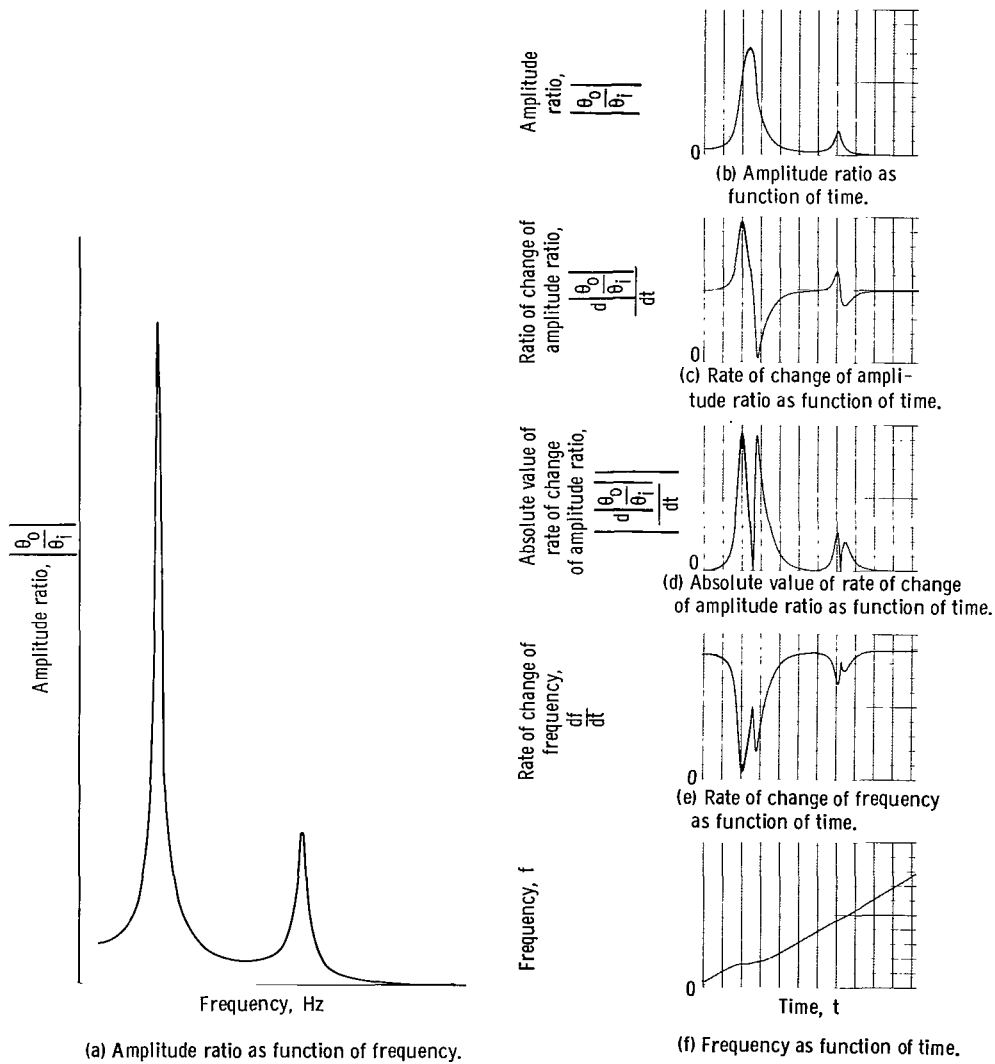


Figure 4. - Typical results for linear variables sweep; doubly resonant system.

The control diagram for this approach is shown in figure 3. The control uses active analog computer components. The choice of  $\tau$  for the approximate differentiator will be discussed later in this section.

The philosophy of this control is that much higher basic sweep rates could be used because the control terms will reduce the sweep rate in the vicinity of resonance. Typical results for this type of sweep are shown for a doubly resonant system in figure 4. Several features in figure 4 might be emphasized. First, frequency (fig. 4(f)) is essentially linear with time, except near resonance (as desired).

Also, because of the distortion of the frequency-time plot, the amplitude-time plot is not equivalent to the amplitude-frequency plot, as it would be for a pure linear sweep with no feedback. Indeed, the distortion has the desirable effect of rounding the tops of the amplitude peaks (against time). This is apparent by comparing the amplitude-time curve with the amplitude-frequency curve (figs. 4(a) to (b)).

Finally, it will be noted that  $\dot{f}$  approaches zero going through the resonance;  $\dot{f} = 0$  is a limiting value, for adding more feedback will usually cause instability. Stability will be considered in detail later in this section.

A large number of variations of the basic scheme are possible; the technique using a basic sweep rate which is logarithmic was also investigated.

## Logarithmic Variable-Sweep Technique

This technique is the same as the linear variable sweep, except that the base sweep rate here is logarithmic. This means that

$$R_L \equiv \frac{1}{t} \log_{10} \frac{f}{f_i}$$

or

$$f = f_i 10^{R_L t}$$

Differentiating (with constant  $R_L$ ) results in

$$\dot{f} = f_i 10^{R_L t} (\log_e 10) R_L = 2.30259 R_L f$$

Hence, this technique is characterized by the fact that the base sweep rate is proportional to the instantaneous frequency. This modification leads to the control law



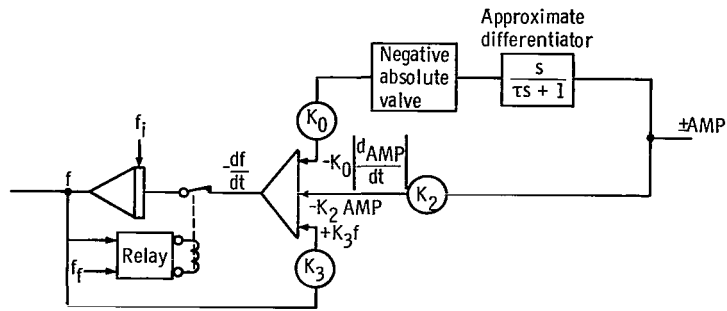


Figure 5. - Logarithmic controller configuration.

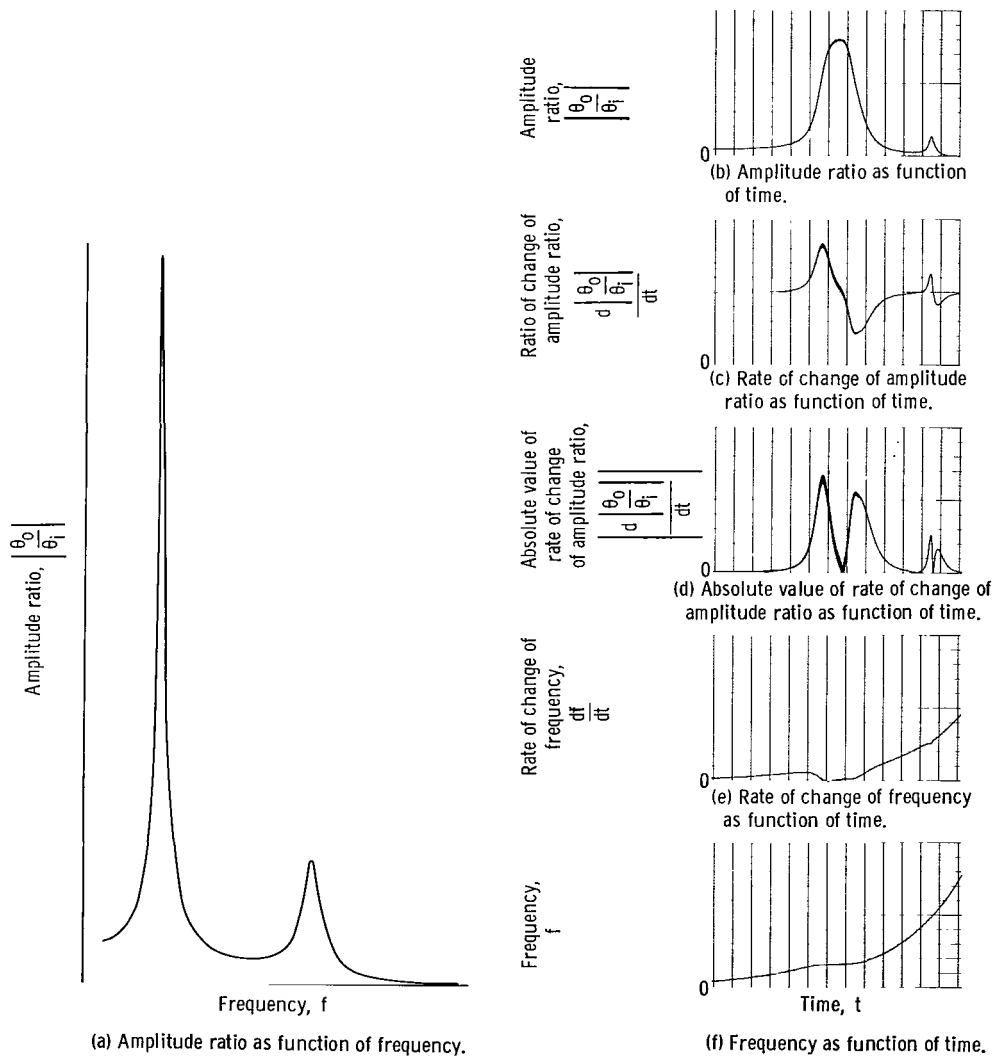


Figure 6. - Typical results for logarithmic variable sweep; doubly resonant system.

$$\frac{df}{dt} = K_3 f - K_0 \left| \frac{d \left| \frac{\theta_o}{\theta_i} \right|}{dt} \right| - K_2 \left| \frac{\theta_o}{\theta_i} \right| \quad (3)$$

and the control diagram of figure 5.

Typical results with this control law for the same system considered previously are presented in figure 6. Comparing  $\dot{f}$  to  $f$  in figure 6(b) reveals the basic characteristic that frequency is proportional to sweep rate. The proportionality is lost near resonance where the control terms are influential. Comparison of  $\dot{f}$  and  $f$  with their counterparts for the linear variable sweep shows the fundamental differences between the two techniques.

### Some Remarks on Technique

The generation of a rate signal is intrinsically a noise amplifying process; for this reason an approximation is generally used for the rate circuitry. The circuit for the approximation used in this study is given in figure 7(a).

The transfer function for this circuit is

$$\frac{v_o(s)}{v_i(s)} = \frac{s}{\tau s + 1} = \frac{s}{(1 - a)s + 1}$$

As  $a$  approaches one, this circuit approaches a pure differentiator. The frequency-response curve for this transfer function is shown in figure 7(b). The approximation deviates from a pure differentiator as the frequency approaches the corner frequency  $f_c$ . In selecting a value of corner frequency, it is important to note that the differentiator must be accurate to the highest frequency content in the amplitude ratio against time plot (i. e.,  $v_{AMP}$  against time in figs. 2, 4(b), and 6(b)); that is, the differentiator need not respond to the highest frequency  $f_f$  in the test range but only to some lower frequency in the vicinity of the corner frequency of the analysis filter. This is because the frequency content of the amplitude ratio against time plot is limited by the analysis filter whose time constant is necessarily below  $f_i$ . This type of consideration for the application which follows (where  $f_i = 10$  Hz and  $f_f = 200$  Hz) led to a choice of 0.79 hertz for the corner frequency of the approximate differentiator. This value appeared to be a satisfactory compromise between noise and accuracy.

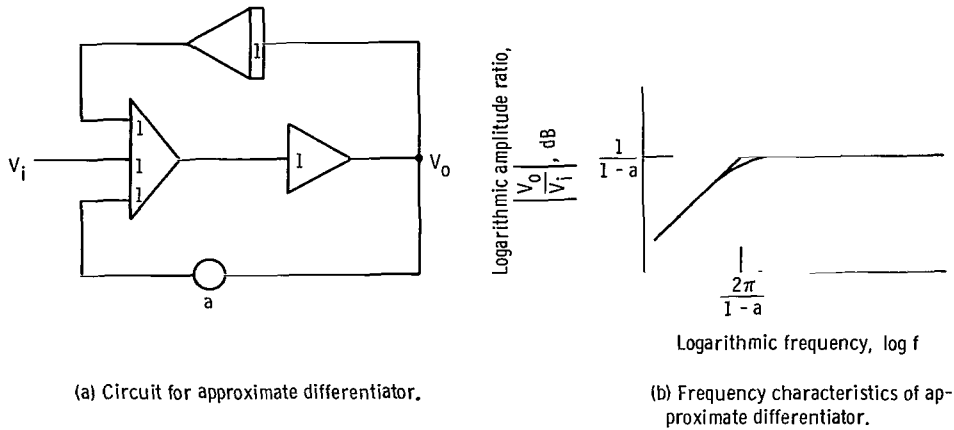


Figure 7. - Approximate differentiator.

## Some Remarks on Stability

If the controller gains  $K_0$  and  $K_2$  (for both controllers, eqs. (2) and (3)) are increased, eventually, the closed-loop system will become unstable. In general, for a sweep with increasing frequency, this instability occurs when

$$\frac{df}{dt} < 0 \quad (5)$$

Occasionally a sweep can be completed when the condition (eq. (5)) is violated momentarily. However, the condition is conservative in that, if it is not violated, the sweep can always be successfully completed. In finding optimum results, the search usually leads as close to  $\dot{f} = 0$  as possible, and it becomes useful to display  $\dot{f}(t)$ .

Considering a linear base sweep, the system will be unstable when

$$\frac{df}{dt} = K_1 - K_0 \left| \frac{d \left| \frac{\theta_o}{\theta_i} \right|}{dt} \right| - K_2 \left| \frac{\theta_o}{\theta_i} \right| < 0 \quad (6)$$

The system will be marginally stable when

$$\frac{df}{dt} = 0$$

Hence,

$$\frac{K_0}{K_1} \left| \frac{d \left| \frac{\theta_o}{\theta_i} \right|}{dt} \right| + \frac{K_2}{K_1} \left| \frac{\theta_o}{\theta_i} \right| = 1 \quad (7)$$

In this form it becomes clear that the maximum magnitude of the controller gains  $K_0$  and  $K_2$  depends on the magnitude of the basic sweep rate  $K_1$ .

Two types of instability have been observed, namely, (1) oscillatory instability where  $\dot{f}$  continuously oscillates in sign and a limit cycle is established so that the sweep cannot be completed and (2) nonoscillatory instability where  $\dot{f}$  goes smoothly to zero and the system is in equilibrium at that value, so that  $f$  would remain at a fixed frequency

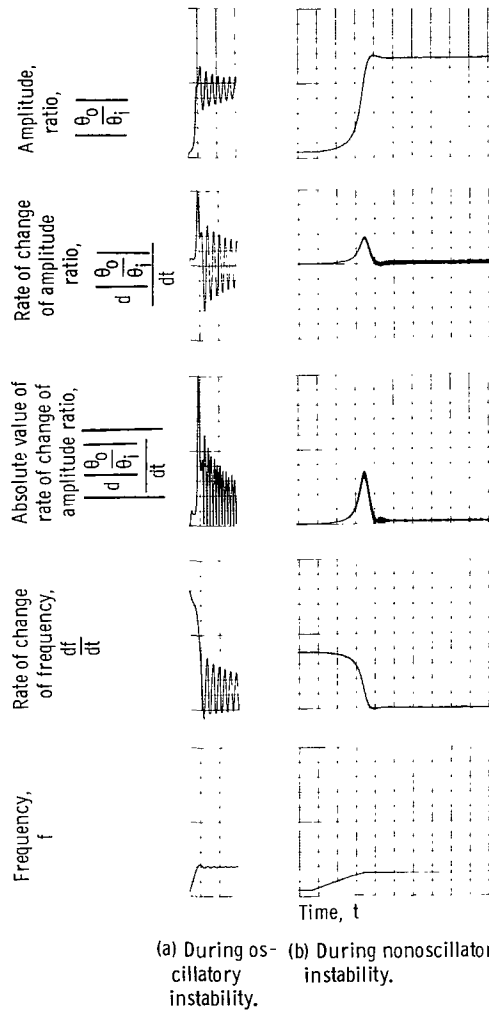


Figure 8. - Typical results as function of time.

value. The first type of instability generally occurs when the rate term is dominant, and the second type is usually caused by too large an amplitude feedback gain  $K_2$ . Typical oscillatory and nonoscillatory instabilities are shown in figure 8. Similar results hold for the logarithmic sweep controller.

## APPLICATION OF TECHNIQUE

In order to demonstrate the time-accuracy relation achievable with these techniques, the scheme was applied to a theoretical system simulated on the analog computer. The system studied was a fourth-order doubly resonant system with the transfer function

$$\frac{\theta_o}{\theta_i} = \frac{1}{\left( \frac{S^2}{\omega_{n1}^2} + \frac{2\xi_1}{\omega_{n1}} S + 1 \right) \left( \frac{S^2}{\omega_{n2}^2} + \frac{2\xi_2}{\omega_{n2}} S + 1 \right)} \quad (8)$$

where the natural frequencies were set to 40 and 115 hertz and the damping numbers were  $\xi = 0.03$  and  $0.01$ , respectively. The system is highly resonant and could represent a control or structural dynamic system.

The criterion of goodness for the control is considered in terms of peak amplitude ratio error. Stated mathematically it is

$$\text{Percent error in peak amplitude ratio} = \frac{\left| \frac{\theta_o}{\theta_i} \right|_{\text{theoretical}} - \left| \frac{\theta_o}{\theta_i} \right|_{\text{sweep}}}{\left| \frac{\theta_o}{\theta_i} \right|_{\text{theoretical}}} \times 100$$

Here  $\left| \frac{\theta_o}{\theta_i} \right|_{\text{theoretical}}$  is the sinusoidal steady state peak amplitude ratio, and  $\left| \frac{\theta_o}{\theta_i} \right|_{\text{sweep}}$  is the peak amplitude ratio associated with sweeping from initial frequency  $f_i$  to final frequency  $f_f$  in time  $T$ . This criterion combines the error due to sweeping and the error associated with the time constant  $\tau_1$  of the measurement system. The measurement error is considered in detail in reference 1 as the wattmeter error.

The schematic diagram for the study is presented in figure 9. The ac coupler circuit and the first-order filter both had 1-hertz corner frequencies. And the approximate differentiator is described by  $a = 0.8$ . The initial frequency  $f_i$  was 10 hertz and the final frequency  $f_f$  was 200 hertz.

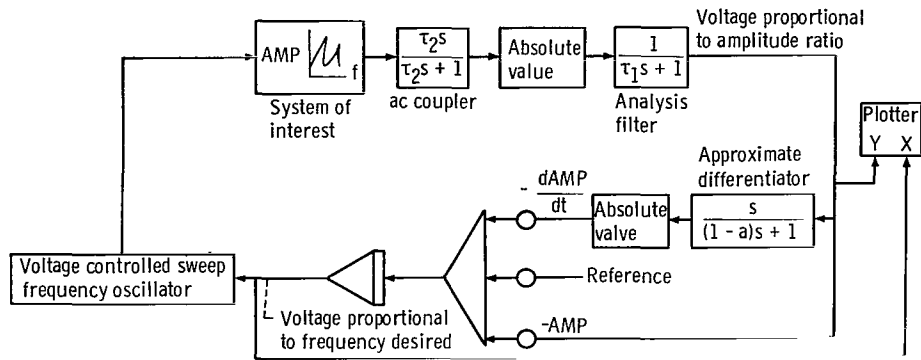


Figure 9. - Schematic diagram of variable sweep rate testing control used for study of doubly resonant system. Linear base sweep.

## Linear Variable-Sweep Technique Results

The performance of the controller, for defining the first resonance, using only rate feedback ( $K_2 = 0$ ) is shown in figure 10(a). The base line of comparison in this figure is the result for a linear sweep with no feedback ( $K_0 = K_2 = 0$ ). At a given base sweep rate  $K_1$  increasing the rate gain  $K_0$  decreases the error at almost no cost in sweep time  $T$  (near vertical lines). This continues until a limiting value at which the system becomes unstable (as defined previously). The dashed line in figure 10(a) represents a locus of neutral stability. Similar results with only amplitude feedback control ( $K_0 = 0$ ) are shown in figure 10(b).

By properly combining both amplitude feedback and rate feedback (fig. 10(c)), a stability locus can be found which is superior to either of the separate control modes. From this plot it is apparent that, if one desires results accurate to, for example, 0.5 percent, the time to accomplish this with a simple linear sweep would be approximately 235 seconds, and the same accuracy could be achieved using the linear variable sweep technique in only 31 seconds (about a 7.5/1 time improvement). Conversely, for a given sweep time, for example, 10 seconds, the error with no control is 35 percent; with the linear variable sweep technique, the error is reduced to 5.2 percent.

The percent error in peak amplitude for the second resonance is shown in figure 11 for the cases of rate only, amplitude ratio only, and combined rate and amplitude ratio feedback. These curves correspond to the stability limits presented separately for the first resonance in figure 10. The control has little influence on the accuracy achieved for the second resonance. The primary reason for this is that the controller gains were set by the more dominant lower frequency resonance.

The controller gains at neutral stability are plotted against basic sweep rate for the pure rate and pure amplitude feedback cases in figure 12. The gain relation associated

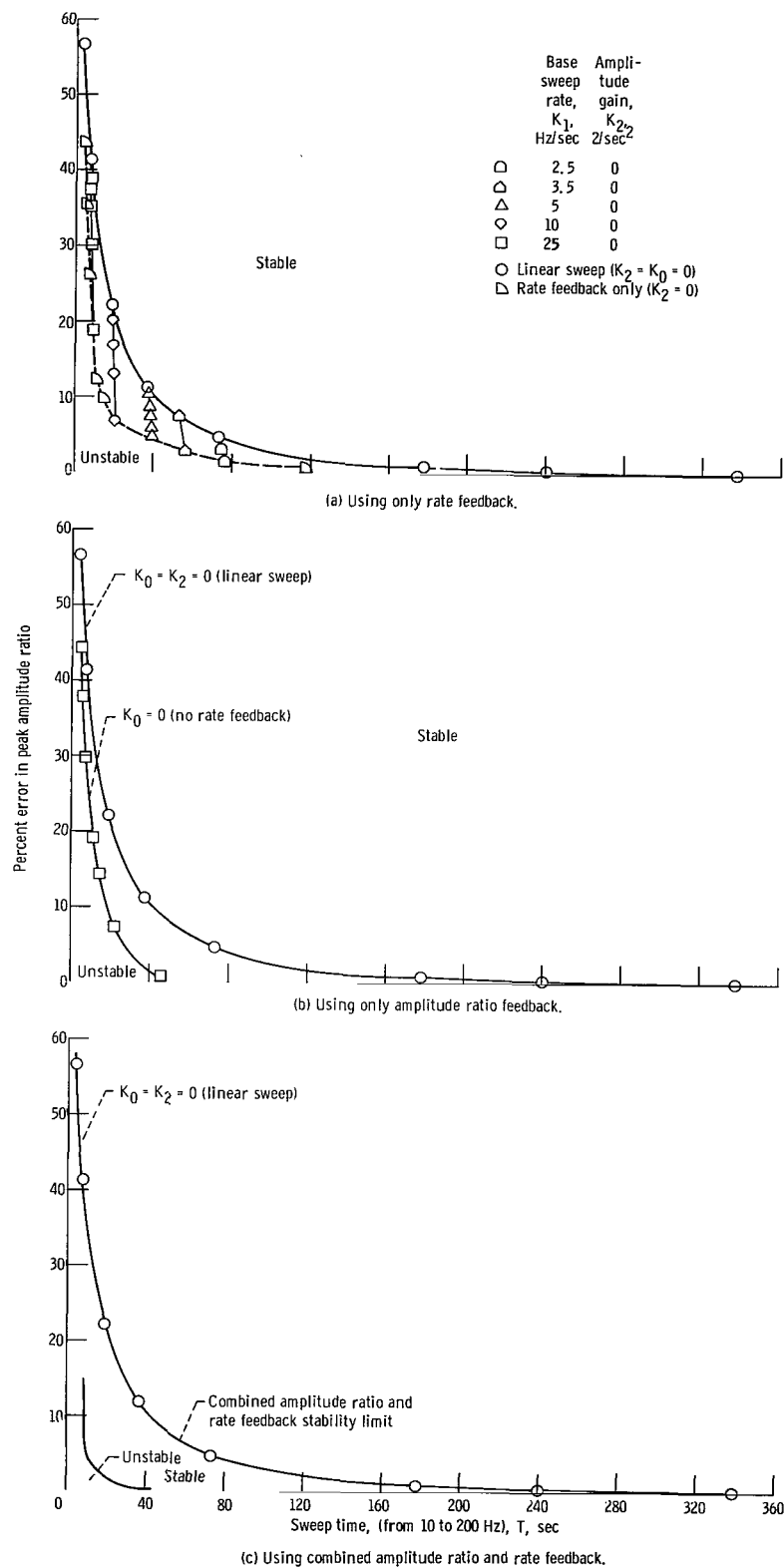


Figure 10. - Percent error in amplitude ratio for linear base sweep technique (for first resonance).

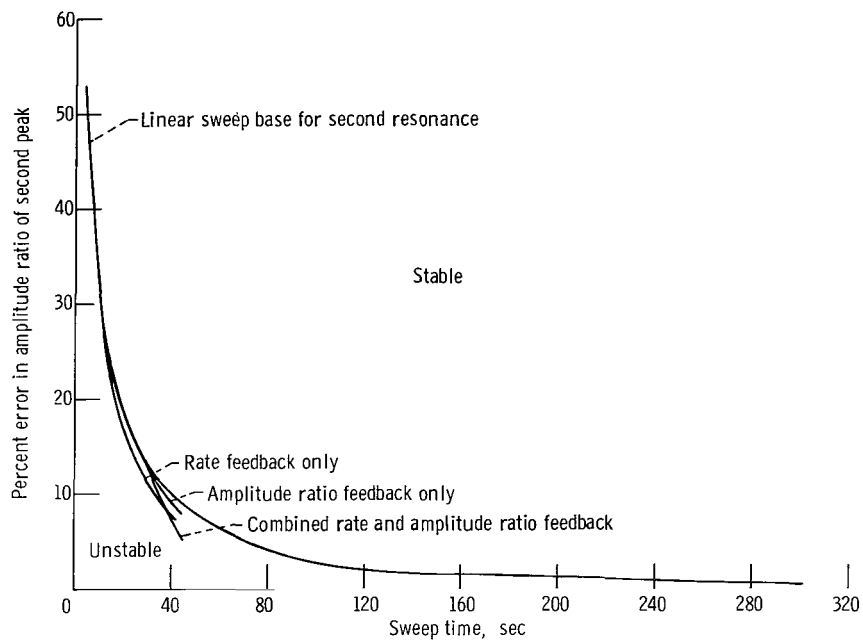


Figure 11. - Percent error in second resonance amplitude ratio using linear base sweep technique.

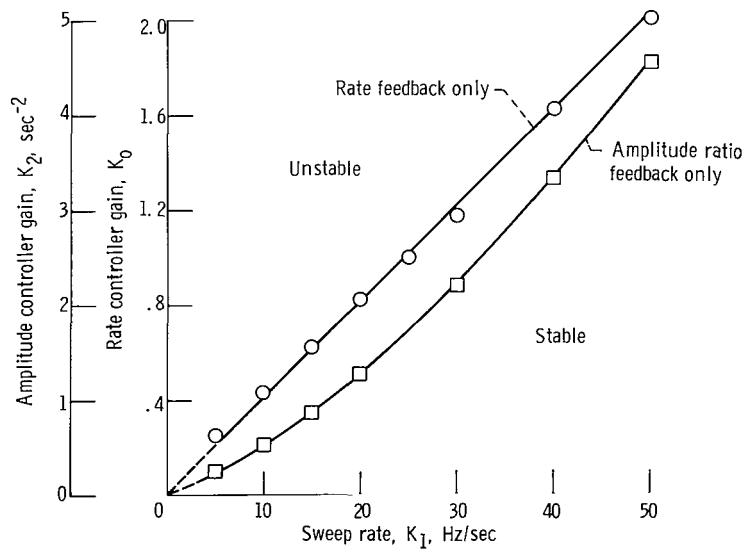


Figure 12. - Controller gains at stability.



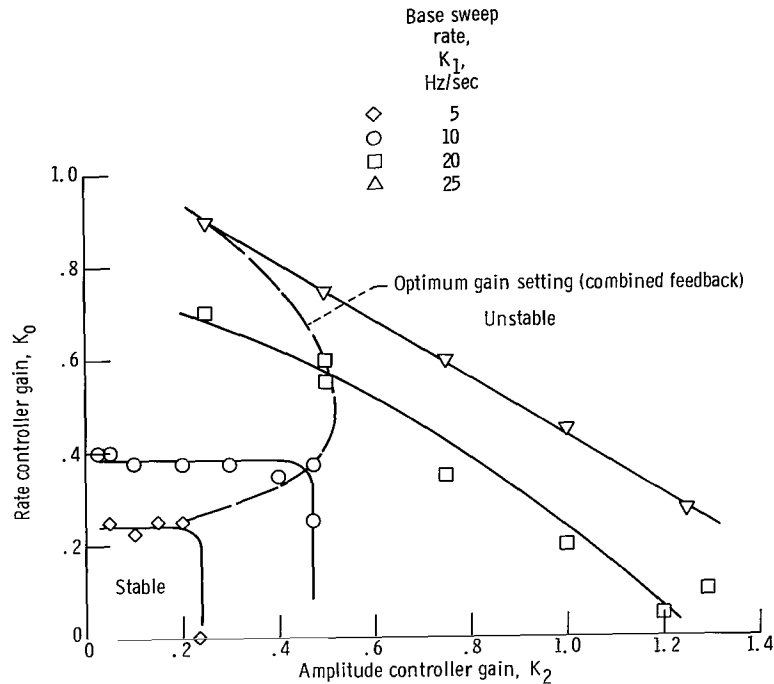


Figure 13. - Neutral stability gains with combined feedback.

with combined feedback stability is shown in figure 13. From the plots of figure 12 it is clear that the amount of control gain that can be applied is almost proportional to the basic sweep rate.

## Logarithmic Variable-Sweep Technique Results

The same system considered in the linear variable-sweep case was used to study performance of the logarithmic base-sweep controller.

The performance for the rate only, amplitude ratio only, and combined rate and amplitude ratio feedbacks for the first and second resonances are plotted in figure 14. Here the base of comparison is a logarithmic sweep with no feedback. The form of the results here are quite similar to linear base sweep (comparison plots will be presented in the next section).

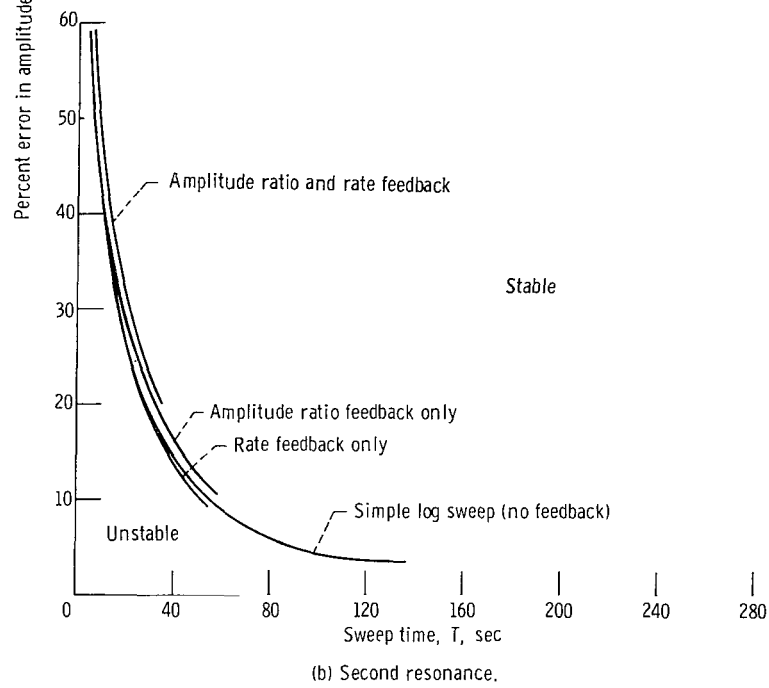
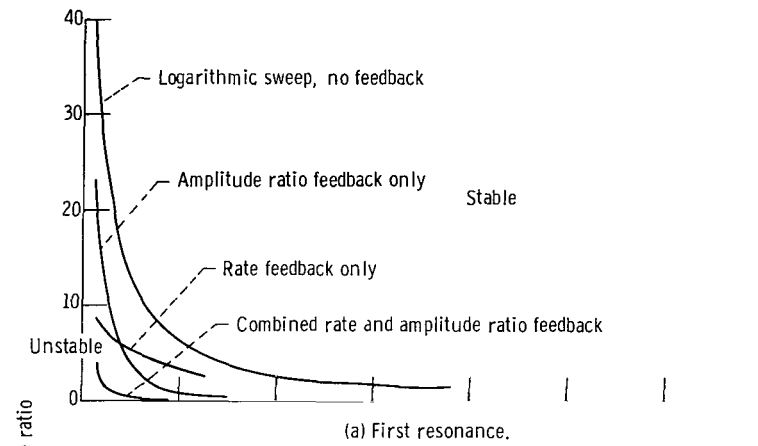


Figure 14. - Percent error in amplitude ratio for logarithmic base sweep.

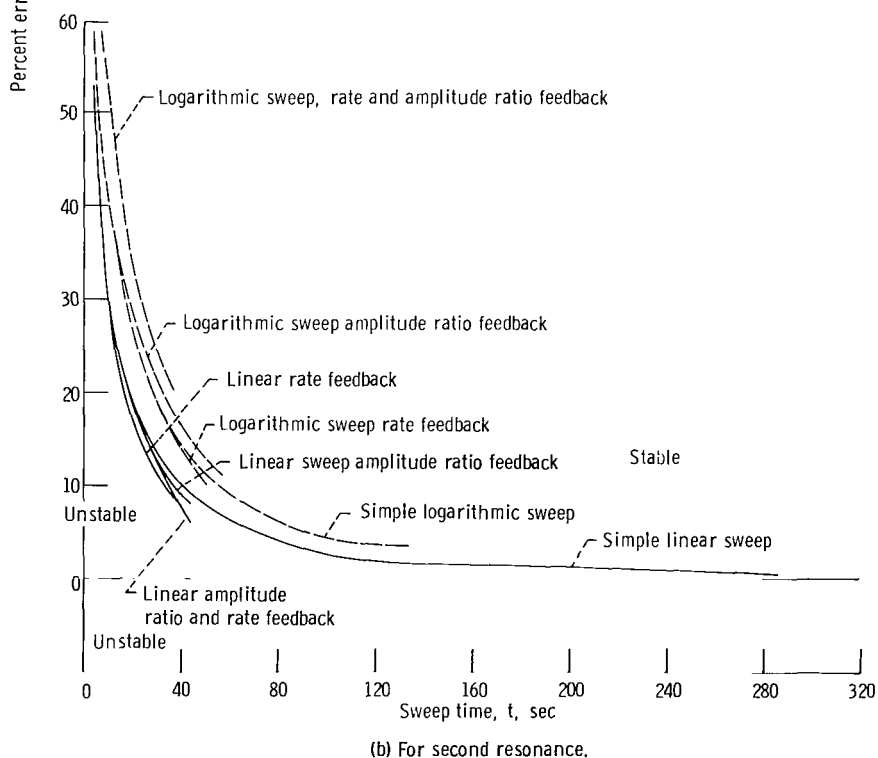
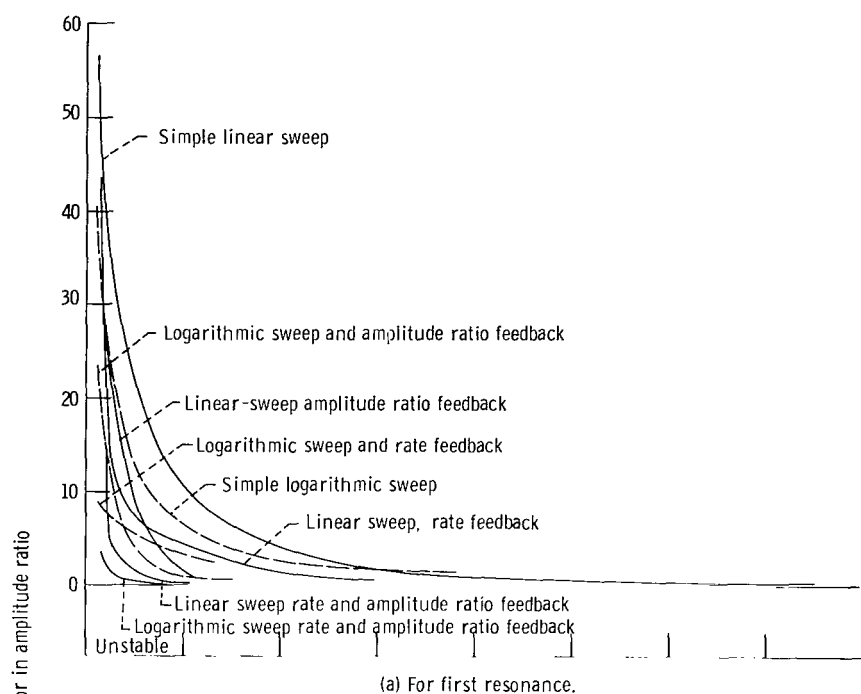


Figure 15. - Comparison of linear and logarithmic sweeps.

## Comparison of Linear and Logarithmic Controllers

The performance of the linear and logarithmic controllers are compared for the first and second resonances in figure 15. The results for the first resonance show the logarithmic base sweep to be superior. Although little control action is being exerted in either technique at the second resonance, the linear base sweep shows up superior. The reason for both of these facts is that the base sweep rate for the logarithmic sweep is lower at the low frequencies and higher at the high frequencies.

In spite of the superiority at low frequencies of the logarithmic sweep, for general testing the linear base would be preferred, since it leads to uniform results across the frequency range; that is, should large resonances occur at both high and low frequencies, they would have about the same time-accuracy quality.

## DISCUSSION

### Frequency-Response Testing

As shown in the preceding sections, the use of the variable-sweep technique offers distinct benefits for frequency response testing. The relative weighting of amplitude ratio feedback and rate feedback to be used for a given class of tests is a philosophical question that must be answered by the user in terms of the application.

By choosing a large amplitude ratio feedback term, the user has said in essence, that he considers resonances of large amplitude to be more important than those of small amplitude. The choice of a large rate feedback says that the level of the resonance is not so important as its sharpness (or quality).

If one considers the effect of pure amplitude ratio feedback on, for example, a first-order system, as frequency increases above the corner frequency, the feedback decreases, thereby increasing sweep rate and error. If, on the other hand, rate feedback was applied (using a logarithmic base sweep), above the corner frequency, the rate of change of amplitude with respect to time would be constant; hence, sweep rate will not be increased.

In an attempt to generalize the results for different systems, a brief study was performed on a second-order system using the linear base sweep technique. For this study natural frequencies, damping numbers, and end frequencies were varied. Although no factor could be found to fully normalize the results, the percent error plotted against the parameter  $\xi f_n T / \Delta f$  (a corrected time parameter) for pure rate feedback fell within a rather narrow range (fig. 16). This plot gives some indication of the results attainable for any given second-order system when compared with the base sweep with no feedback.

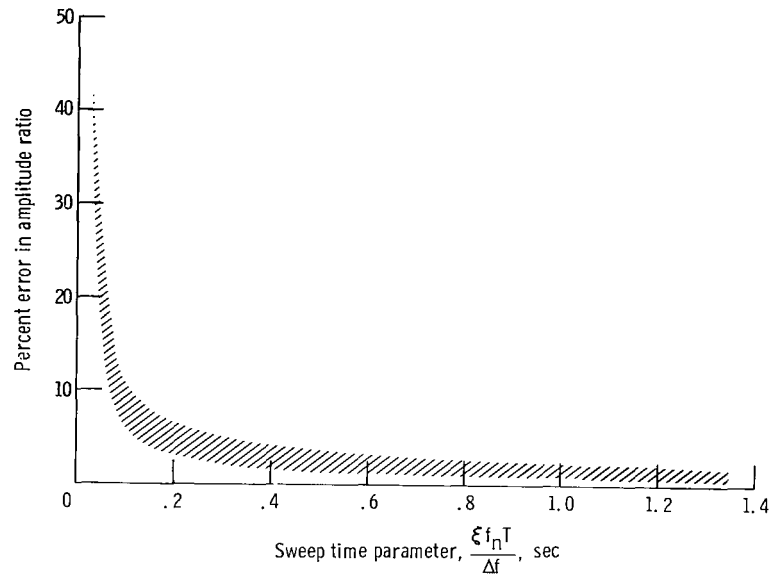


Figure 16. - Normalized performance of linear variable sweep system with rate feedback.

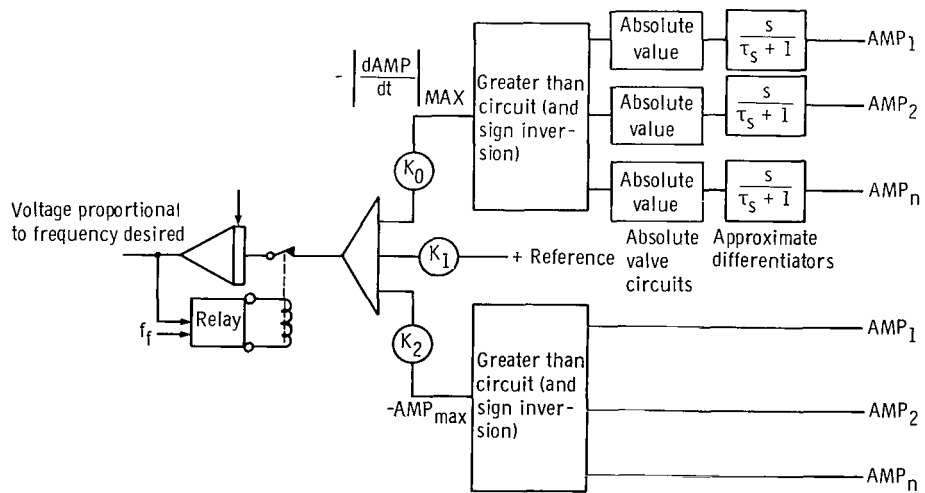


Figure 17. - Modification of linear controller configuration for multiple control variable operation.

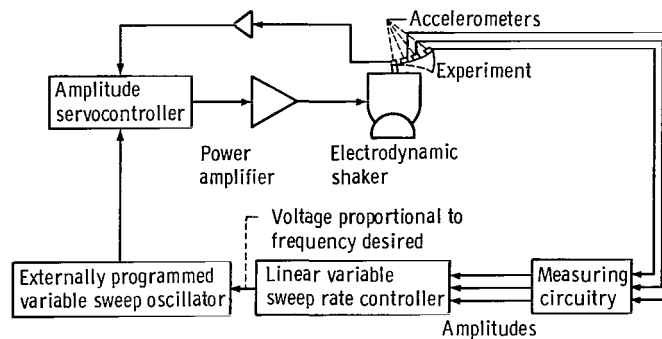


Figure 18. - Test configuration for variable sweep rate vibration testing.

## Vibration Testing

The variable-sweep technique can be readily applied to vibration testing. For application to large scale systems such as a rocket vehicle, however, it would be desirable to close the loop not on a single variable but on several variables; that is, several variables would be selected as important variables indicating resonance. Then, for the amplitude ratio feedback the largest of these would be the active signal. Similarly, for the rate feedback the variable with the largest rate would be the control signal. For the linear base technique, then, the diagram of figure 3 would be modified as shown in figure 17 to handle multiple control variables. A typical test setup for use of the technique for vibration testing is shown in figure 18.

## SUMMARY OF RESULTS

A technique has been established to improve the quality and acquisition time of frequency response and vibration data. The technique involves sweeping at a variable sweep rate such that the sweep rate is slow during periods of dynamic importance and fast otherwise. Methods of sensing these important periods (resonances) were studied. The technique was demonstrated for a computer simulated system.

Significant improvement in sweep time for a required accuracy of results, or in result accuracy for a given sweep time, can be effected using the variable sweep technique. For the linear base sweep a 7.5 to 1 time improvement for a 0.5-percent accuracy on a known system was shown. Also, for a 10-second sweep on the same system the accuracy was improved from 35 to 5.2 percent.

The limiting factor in the improvements that can be made is system stability. An easily used indicator of stability is the time derivative of frequency  $f$ . By monitoring  $\dot{f}$  during a sweep and maintaining the criteria  $\dot{f} > 0$  by setting control gains appropriately, the unstable condition can be avoided.

Two techniques were studied; a logarithmic base sweep and a linear base sweep. The logarithmic base sweep gave better results for the low-frequency end of the system studied. However, the linear-base technique is recommended because it tends to give more uniform results over the frequency range.

The criterion of goodness used in the study was a simple one, namely, percent variation in peak magnitude. However, this is indicative of overall quality and does demonstrate what can be achieved with the techniques.

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, September 10, 1970,  
720-03.

## APPENDIX - SYMBOLS

|            |  |             |  |
|------------|--|-------------|--|
| AMP        | amplitude ratio  | $\tau_2$    | ac coupler time constant, sec                                  |
| a          | constant in eq. (4)  | v           | voltage, V   |
| f          | frequency, Hz  | $\omega_n$  | natural frequency, rad/sec                                     |
| $\Delta f$ | frequency range = $f_f - f_i$ , Hz                           | $\xi$       | damping ratio number   |
| $K_0$      | rate gain (see eqs. (2) and (3)),<br>1/sec                   | $\theta$    | variable   |
| $K_1$      | base sweep rate (see eq. (2)),<br>1/sec <sup>2</sup>         | Subscripts: |  |
| $K_2$      | amplitude gain (see eqs.<br>(2) and (3)), 1/sec <sup>2</sup> | AMP         | amplitude ratio  |
| $K_3$      | 2.30259 $R_L$ , 1/sec  | c           | corner   |
| $R_L$      | logarithm sweep rate, decade/sec                             | i           | initial - for frequencies                                      |
| s          | Laplace operator   | i           | input - for transfer functions                                 |
| T          | sweep time from $f_i$ to $f_f$ , sec                         | f           | final  |
| t          | time, sec  | n           | number of channels   |
| $\tau$     | time constant, sec   | o           | output   |
| $\tau_1$   | filter time constant, sec                                    | max         | maximum  |
|            |  | .           | quantities represent differentia-<br>tion with respect to time |



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